SIGNED *q*ROMAN *K*-DOMINATING FUNCTION OF COMPOSITION OF GRAPHS

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ABSTRACT: *Continuing the study of Roman dominating function, Ahangar et al. (2014) introduced and studied the signed Roman dominating function. A generalization of the signed Roman dominating function is the Roman k-dominating function which was presented and studied by Henning and Volkmann (2015). We present in this study an offshoot of the signed Roman k-dominating function, called the signed qRoman k-dominating function.*
Let k be a positive integer. A signed qRoman k-dominating function on G is the function $f: V \rightarrow -1,1,2$ *with the following two properties: (1)* $f[u]_q = \sum_{v \in N(u)} f(v) \geq k$ *for all* $u \in V$*; and (2) for every vertex v with* $f(v) = -1$*, there exists a vertex w with* $f(w) = 2$ *such that* $vw \in E$*. The weight of a signed qRoman k-dominating function is* $w(f) = \sum_{v \in V} f(v)$*.The signed qRoman k-dominating number of G, denoted by* $\gamma^k_{qR}(G)$*, is the minimum weight of a signed qRoman k-dominating function on G.*
This study gave the signed qRoman 2-domination number of: crisscross graphs and tensor product of graphs.
Keywords: Signed *q*Roman *k*-Dominating Function, Criss-cross Graphs, Tensor Product of Graphs.

INTRODUCTION

A *signed Roman k-dominating function* in a simple graph $G = (V,E)$ is a function $f: V \rightarrow \{-1,1,2\}$ satisfying two properties. *(1)* $f[u]_q = \sum_{v \in N(u)} f(v) \geq k$ for all $u \in V$; and *(2)* for every $u \in V$ with $f(v) = -1$, there exists $y \in V$ such that $vy \in E$ and $f(y) = 2$. The *weight* of a signed *q*Roman *k*-dominating function is $w(f) = \sum_{v \in V} f(v)$. The signed *q*Roman *k*-dominating number of *G*, denoted by $\gamma^k_{qR}(G)$, is the minimum weight of a signed *q*Roman *k*-dominating function on *G*. This concept was introduced and studied by Henning and Volkmann (2015). The special case (the case *k=1*) came out first and was presented and studied by Ahangar et. Al. (2014). The case *k=2* was also studied by Adanza et. al. (2015).

This study is an off shoot of the *signed Roman k-dominating function* called the signed *q*Roman *k*-dominating number of crisscross graphs and tensor product of graphs. A *signed qRoman 2-dominating function* on *G* is a function $f: V \rightarrow \{-1,1,2\}$ satisfying two properties. *(1)* $f[u]_q = \sum_{v \in N(u)} f(v) \geq k$ for all $u \in V$; and *(2)* for every vertex *v* with $f(v) = -1$, there exists a vertex *w* with *f(w)=2* such that $vw \in E$. The subscript *q* was added to differentiate the two functions.

The two definition differ in the first condition. The signed Roman *k*-dominating function requires that $f[u] = \sum_{v \in N[u]} f(v) \geq k$ for all $u \in V$ while the signed *q*Roman 2-dominating function only requires $f[u]_q = \sum_{v \in N(u)} f(v) \geq k$.

The *weight* of a signed *q*Roman *k*-dominating function is $w(f) = \sum_{v \in V} f(v)$. The signed *q*Roman *k*-dominating number of *G*, denoted by $\gamma^k_{qR}(G)$, is the minimum weight of a signed *q*Roman *k*-dominating function on *G*.

Let $G = (V,E)$ be a graph and $u \in V$. The *open neighborhood* of *u*, denoted by *N(u)*, consists of all vertices $y \in V$ such that $uv \in E$. The *closed neighborhood* of *u*, denoted by *N[u]*, is given by $N[u] = N(u) \cup \{u\}$.

The path $P_n = (v_1, v_2, ..., v_n)$ is the graph with distinct vertices $v_1, v_2, ..., v_n$ and edges $v_1v_2, v_2v_3, ..., v_{n-1}v_n$.

The cycle $C_n = [v_1, v_2, ..., v_n], n \geq 3$, is the graph with vertices $v_1, v_2, ..., v_n$ and edges $v_1v_2, v_2v_3, ..., v_{n-1}v_n$, v_nv_1. If n is even, then C_n is called an *even cycle*; if n is odd then Loading... is an *odd cycle.*

A *complete* graph of order *n*, denoted by K_n, is the graph in which every pair of distinct vertices are adjacent.

The *composition G[H]* of two graphs G and H is the graph with vertex set $V(G[H] = V(G) \times V(H)$ and edge set *E(G[H])* satisfying the following condition: $(u_1,u_2)(v_1,v_2) \in E(G[H])$ if and only if either $u_1v_1 \in E(G)$ or, $u_1 = u_2$ and $v_1v_2 \in E(H)$.

The composition of $P_3[P_2]$ of graphs $P_3 = [a, b, c]$ and $P_2[x, y]$ is shown in Figure 1.

Let *k* be a positive integer. A *signed qRoman k-dominating function* on *G* is a function $f: V \to \{-1,1,2\}$ satisfying two properties. *(1)*

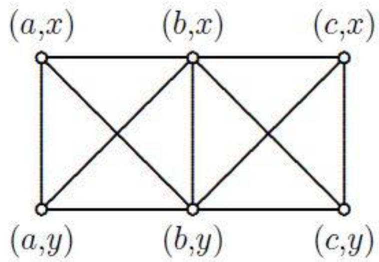

Figure 1: The graph $P_3[P_2]$

$f[u]_q = \sum_{v \in N(u)} f(v) \geq k$ for all $u \in V$; and *(2)* for every vertex *v* with $f(v) = -1$, there exists a vertex *w* with $f(w) = 2$ such that $vw \in E$. The *weight* of a signed *q*Roman *k*-dominating function is $w(f) = \sum_{v \in V} f(v)$. The signed *q*Roman *k*-dominating number of *G*, denoted by $\gamma^k_{qR}(G)$, is the minimum weight of a signed *q*Roman *k*-dominating function on *G*.

Let $V = \{a,b,c,d,e,f\}$ and $E = \{ab,bc,cd,de,ef,af\}$. Then the function $f: V \to \{-1,1,2\}$ given by $a \longmapsto 1, b \longmapsto 1, c \longmapsto 1, d \longmapsto 1, e \longmapsto 1$ and $f \longmapsto 1$ is a signed *q*Roman 2-domination number of *G*. Note that *f* is a minimum signed *q*Roman 2-domination number. Hence $\gamma^k_{qR}(G) = 6$.

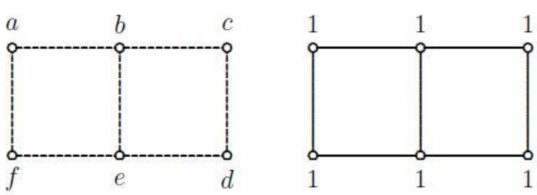

Figure 2: A graph G

PRELIMINARY RESULTS

This section presents the results of our study.

Signed *q*Roman 2-domination Number of the Composition of Paths, Cycles and Complete Graphs.

In this section we gave some upper bounds for the signed *q*Roman *k*-domination number of composition of graphs.

Theorem 1 *Let P_n and P_m be paths of order n and m, respectively. Then*

$$\gamma^2_{qR}\left(P_n\left[P_m\right]\right) \leq \begin{cases} 4n, & if\ m\ is\ even \\ 5n, & if\ m\ is\ odd \end{cases}$$

Proof: Let P_n and P_m be paths of order *n* and *m*, respectively. Define $f: V\left(P_n\left[P_m\right]\right) \to \{-1,1,2\}$ by

$$f(i,j) = \begin{cases} 2, & if\ j = 1, m \\ 1, & if\ j\ is\ even,\ 2 \leq j \leq m-1 \\ -1, & if\ j\ is\ odd,\ 3 \leq j \leq m-1 \end{cases}$$

Then *f* is a signed *q*Roman 2-dominating function. Thus,

$$\gamma^2_{qR}\left(P_n\left[P_m\right]\right) \leq \begin{cases} 4n, & if\ m\ is\ even \\ 5n, & if\ m\ is\ odd. \square \end{cases}$$

Theorem 2 *Let P_n be path of order n and C_m be a cycle of order m. Then*

$$\gamma^2_{qR}\left(P_n\left[C_m\right]\right) \leq \begin{cases} 4n, & if\ m\ is\ even \\ 5n, & if\ m\ is\ odd. \end{cases}$$

Proof: Let P_n be a path of order *n* and C_m be a cycle of order *m*. Define $f: V\left(P_n\left[C_m\right]\right) \to \{-1,1,2\}$ by

$$f(i,j) = \begin{cases} 2, & if\ j = 1,m \\ 1, & if\ j\ is\ even,\ 2 \leq j \leq m-1 \\ -1, & if\ j\ is\ odd,\ 3 \leq j \leq m-1. \end{cases}$$

Then *f* is a signed *q*Roman 2-dominating function. Thus,

$$\gamma^2_{qR}\left(P_n\left[C_m\right]\right) \leq \begin{cases} 4n, & if\ m\ is\ even \\ 5n, & if\ m\ is\ odd. \square \end{cases}$$

Theorem 3 *Let C_n and C_m be cycles of order n and m, respectively. Then*

$$\gamma^2_{qR}\left(C_n\left[C_m\right]\right) \leq \begin{cases} 4n, & if\ m\ is\ even \\ 5n, & if\ m\ is\ odd. \end{cases}$$

Proof: Let C_n and C_m be a cycle of order *n* and *m*, respectively. Define $f: V\left(C_n\left[C_m\right]\right) \to \{-1,1,2\}$ by

$$f(i,j) = \begin{cases} 2, & if\ j = 1,m \\ 1, & if\ j\ is\ even,\ 2 \leq j \leq m-1 \\ -1, & if\ j\ is\ odd,\ 3 \leq j \leq m-1. \end{cases}$$

Then *f* is a signed *q*Roman 2-dominating function. Thus,

$$\gamma^2_{qR}\left(C_n\left[C_m\right]\right) \leq \begin{cases} 4n, & if\ m\ is\ even \\ 5n, & if\ m\ is\ odd. \square \end{cases}$$

Theorem 4 *Let C_n be cycle of order n and P_m be a path of order m. Then*

$$\gamma^2_{qR}\left(C_n\left[P_m\right]\right) \leq \begin{cases} 4n, & if\ m\ is\ even \\ 5n, & if\ m\ is\ odd. \end{cases}$$

Proof: Let C_n be a cycle of order *n* and P_m be a path of order *m*. Define $f: V\left(C_n\left[P_m\right]\right) \to \{-1,1,2\}$ by

$$f(i,j)=\begin{cases} 2, \text{ if } j=1,m \\ 1, \text{ if } j \text{ is even}, 2\le j\le m-1 \\ -1, \text{ if } j \text{ is odd}, 3\le j\le m-1. \end{cases}$$

Then *f* is a signed *q*Roman 2-dominating function. Thus,

$$\gamma^2_{qR}\left(C_n\left[P_m\right]\right)\le\begin{cases} 4n, \text{ if } m \text{ is even} \\ 5n, \text{ if } m \text{ is odd}.\square \end{cases}$$

Theorem 5 *Let* K_n *be complete graph of order n and* P_m *be a path of order m. Then*

$$\gamma^2_{qR}\left(K_n\left[P_m\right]\right)\le\begin{cases} 2n, \text{ if } m \text{ is even} \\ 3n, \text{ if } m \text{ is odd}. \end{cases}$$

Proof: Let K_n be a complete graph of order *n*, and P_m be a path of order *m*. Define $f:V\left(K_n\left[P_m\right]\right)\to\{-1,1,2\}$ by

$$f(i,j)=\begin{cases} 2, \text{ if } (i,j)=(1,1),(1,m),(n,1),(n,m) \\ 1, \text{ if } j \text{ is odd and } 2\le i\le n-1; \text{ or, } i=1,n \text{ and } j \text{ is odd} \\ \quad \text{with } 3\le j\le m-2 \text{ if } m \text{ is odd and } 3\le j\le m-1 \text{ if } m \text{ is even} \\ -1, \text{ if } j \text{ is even and } 2\le i\le n-1; \text{ or, } i=1,n \text{ and } j \text{ is even} \\ \quad \text{with } 2\le j\le m-1 \text{ if } m \text{ is odd and } 2\le j\le m-2 \text{ if } m \text{ is even} \end{cases}$$

Then *f* is a signed *q*Roman 2-dominating function. Thus,

$$\gamma^2_{qR}\left(K_n\left[P_m\right]\right)\le\begin{cases} 2n, \text{ if } m \text{ is even} \\ 3n, \text{ if } m \text{ is odd}.\square \end{cases}$$

Theorem 6 *Let* K_n *be complete graph of order n and* C_m *be a cycle of order m. Then*

$$\gamma^2_{qR}\left(K_n\left[C_m\right]\right)\le\begin{cases} 2n, \text{ if } m \text{ is even} \\ 3n, \text{ if } m \text{ is odd}. \end{cases}$$

Proof: Let K_n be a complete graph of order *n*, and C_m be a cycle of order *m*. Define $f:V\left(K_n\left[C_m\right]\right)\to\{-1,1,2\}$ by

$$f(i,j)=\begin{cases} 2, \text{ if } (i,j)=(1,1),(1,m),(n,1),(n,m) \\ 1, \text{ if } j \text{ is odd and } 2\le i\le n-1; \text{ or, } i=1,n \text{ and } j \text{ is odd} \\ \quad \text{with } 3\le j\le m-2 \text{ if } m \text{ is odd and } 3\le j\le m-1 \text{ if } m \text{ is even} \\ -1, \text{ if } j \text{ is even and } 2\le i\le n-1; \text{ or, } i=1,n \text{ and } j \text{ is even} \\ \quad \text{with } 2\le j\le m-1 \text{ if } m \text{ is odd and } 2\le j\le m-2 \text{ if } m \text{ is even} \end{cases}$$

Then *f* is a signed *q*Roman 2-dominating function. Thus,

$$\gamma^2_{qR}\left(K_n\left[C_m\right]\right)\le\begin{cases} 2n, \text{ if } m \text{ is even} \\ 3n, \text{ if } m \text{ is odd}.\square \end{cases}$$

Signed *q*Roman 2-domination Number of the Union of Graphs.

In this section we get the signed *q*Roman *k*-domination number of the union of graphs.

Theorem 7 *Let G and H be a graphs and* $k\in\mathbb{N}$. *Then* $\gamma^k_{qR}(G\cup H)=\gamma^k_{qR}(G)+\gamma^k_{qR}(H)$ *if* $\gamma^k_{qR}(G)$ *and* $\gamma^k_{qR}(H)$ *exists.*

Proof: Let $G=\left(V_1,E_1\right)$ and $H=\left(V_2,E_2\right)$ be graphs. Suppose that $\gamma^k_{qR}(G)$ and $\gamma^k_{qR}(H)$ exists. Let f_1 and f_2 be signed *q*Roman *k*-dominating functions with $w\left(f_1\right)=\gamma^k_{qR}(G)$ and $w\left(f_2\right)=\gamma^k_{qR}(H)$. Define $f:V_1\cup V_2\to\{-1,1,2\}$ by

$$f(u)=\begin{cases} f_1(u), \text{ if } u\in V_1 \\ f_2(u), \text{ if } u\in V_2. \end{cases}$$

Then clearly *f* is a signed *q*Roman *k*-dominating function of $(G\cup H)$. Since $w(f)=w\left(f_1\right)+w\left(f_2\right)$, we must have $\gamma^k_{qR}(G\cup H)\le\gamma^k_{qR}(G)+\gamma^k_{qR}(H)$. It can be shown that $\gamma^k_{qR}(G\cup H)$ cannot be less than $\gamma^k_{qR}(G)+\gamma^k_{qR}(H)$. Hence, $\gamma^k_{qR}(G\cup H)=\gamma^k_{qR}(G)+\gamma^k_{qR}(H).\square$

CONCLUSION

The results of signed *q*Roman *k*-domination maybe used in the design of communication networks. Furthermore, the results of this study will give a more complete understanding on the concept signed Roman *k*-domination in graphs. Since some interesting results are revealed in this study, I recommended that the signed *q*Roman *k*-domination number be investigated further such as composition of arbitrary paths, cycles and complete graphs.

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REFERENCES:

[1] J. Adanza and M. Baldado. Signed qRoman k-Domination Number of the Join and Corona of Graphs. Mathematics Department (Negros Oriental State University, http://dx.doi.org/10.12988/arms.2015.57484

[2] H. Ahangar, M. Henning, Y. Zhao, C. Lowenstein, and V. Samodivkin. Signed Roman domination in graphs. *J. Comb. Optim.*, 27 (2014), 241-255.

[3] A. Behtoei, E. Vatandoos, and F.A. Abad. The signed Roman Domination Number of Some Graphs, *arXiv:1411.7208v1 [math.CO]* 26, Nov 2014.

[4] E. Cockayne, P. Dreyer Jr., S.M Hedetniemi, and S.T. Hedetniemi. Roman Domination in Graphs, *Discrete Mathematics*, 278(2004), 11-22.

[5] H. Fernau. Roman Domination: a parameterized perspenctive, *Int J Computmath 85,* (2008), 25-38.

[6] M. Kattimani and N. Hemalatha. Roman Domination Number of a Graph, *International Journal of Scientific and Research Publication,* Volume 6, Issue 6 2016.

[7] P. Palvic and J. Zerovnik. Roman Domination Number of the Cartesian Products, *The Electronic Journal of Combinatorics* 19 (3) (2012), p. 19.

[8] Fu Xuelang, Y. Yuansheng and J. Boaqi. Roman Domination in Regular Graphs, *Discreet Mathematics*, 309 (2009), 1528 – 1537